

Fuelling quasars with hot gas

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ABSTRACT

We consider a model for quasar formation in which massive black holes are formed and fuelled largely by the accretion of hot gas during the process of galaxy formation. In standard hierarchical collapse models, objects about the size of normal galaxies and larger form a dense hot atmosphere when they collapse. We show that if such an atmosphere forms a nearly “maximal” cooling flow, then a central black hole can accrete at close to its Eddington limit. This leads to exponential growth of a seed black hole, resulting in a quasar in some cases. In this model, the first quasars form soon after the first collapses to produce hot gas. The hot gas is depleted as time progresses, mostly by cooling, so that the accretion rate eventually falls below the threshold for advection-dominated accretion, at which stage radiative efficiency plummets and any quasar turns off. A simple implementation of this model, incorporated into a semi-analytical model for galaxy formation, over-produces quasars when compared with observed luminosity functions, but is consistent with models of the X-ray Background which indicate that most accretion is obscured. It produces few quasars at high redshift due to the lack of time needed to grow massive black holes. Quasar fuelling by hot gas provides a minimum level, sufficient to power most quasars at redshifts between one and two, to which other sources of fuel can be added. The results are sensitive to feedback effects, such as might be due to radio jets and other outflows.

Key words: galaxies:formation – quasars:general

1 INTRODUCTION

It is generally accepted that quasars are the result of accretion onto massive black holes residing in the nuclei of normal galaxies. Rees (1984) has argued that a black hole is likely to form at the centre of almost any galaxy, so that the main issue for quasar formation is how the black hole is fuelled. Models for quasar evolution must account for the time dependence of the quasar luminosity function, particularly its peak at $z \sim 1.5$ and subsequent decline (*e.g.* Boyle, Shanks & Peterson, 1988), and also for the formation of the massive black holes and their remnants that reside in the nuclei of many nearby galaxies (*e.g.* Ford et al 1997; Magorrian et al 1998).

A variety of models has been proposed for the fuelling of quasars, most of which rely on making interstellar gas fall close to the nucleus where it joins an accretion disc (*e.g.* Shlosman, Begelman & Frank 1990). In this paper we consider the possibility that the main source of fuel is the hot interstellar medium formed during the collapse of larger galaxies. Provided that the angular momentum of the gas is not too large, a nuclear black hole can grow by Bondi accretion and, if the gas temperature is close to the virial

temperature, its growth time is controlled largely by the density of the hot gas. In section 3 we show that, soon after the collapse of a protogalaxy, this can be large enough to make the accretion rate of a nuclear black hole comparable to the Eddington rate.

It is the protogalaxies with the largest spheroids, *i.e.* large elliptical protogalaxies, that contain most hot gas (Nulsen & Fabian 1997), making them the best hosts for quasars in this model. Thus, the observation that quasars reside in elliptical hosts (McLure et al 1998) is consistent with them being fuelled by hot gas. Based on the presence of companions close to many quasars, it has been argued that gravitational interaction plays a significant role in the quasar phenomenon (*e.g.* Bahcall et al 1997). However, we note that this may also be interpreted as indicating recent collapse. As outlined below, the supply of hot gas is expected to be greatest immediately after a collapse and to decrease with time, so this would also be consistent with fuelling quasars by hot gas. Of course, active galactic nuclei, including quasars, may obtain their fuel from a variety of sources. We consider that hot gas provides a minimum fuelling rate, which can be supplemented by cold gas, if available.

Cooling depletes the hot gas throughout the galaxy, so that the nuclear accretion rate decreases with time after a protogalaxy collapses. The depletion of the hot gas does not however simply explain the lack of luminous quasars at the current epoch. The central black holes in nearby elliptical galaxies are still immersed in accretable hot gas, yet generally have low accretion luminosities (Fabian & Canizares 1988; Di Matteo & Fabian 1997). As an example, one of the best candidates for a massive nuclear black hole is M87 (Harms et al 1994; Marconi et al 1997). A solution to this problem has been argued by Di Matteo & Fabian (1997) and, specifically for M87, by Reynolds et al (1996) in which the accretion flow becomes advection-dominated (i.e. an ADAF, Narayan & Yi 1995), so having a low accretion efficiency. We adopt that hypothesis here and assume that when the nuclear accretion rate falls below the threshold for ADAF formation most quasars fade rapidly (Fabian & Rees 1995; Yi 1996).

In section 4 we describe the incorporation of a simple version of this model for quasar formation into a semi-analytical model for galaxy formation (Nulsen & Fabian 1997; see also Haehnelt & Rees 1993 for models relating quasar evolution to galaxy formation). This is used to show that the model can account for the broad features of the history of quasars in section 5. Section 6 has a brief discussion of feedback on the model, particular that due to Compton cooling. In section 7 we discuss the limitations of the semi-analytical model for quasar formation and some predictions of the model. Our conclusions are summarized in section 8.

2 ANGULAR MOMENTUM AND THE FEEDING OF A NUCLEAR BLACK HOLE

Shlosman et al (1990) discuss the issues of getting a large mass of gas to accrete into the nucleus of a galaxy. Almost all galaxies have appreciable net rotation, so that the main difficulty is the dissipation of angular momentum, essentially all of which must be dissipated in order for gas to accrete into a nuclear black hole. The total accreted matter should also account for (most of) the mass of the nuclear black hole, which exceeds $10^9 M_\odot$ in many cases (Ford et al 1997; Magorrian et al 1998), so that the gas needs to be drained from a large region around the nucleus. The larger this region, the greater the difficulty of dissipating angular momentum.

According to the standard argument, the effective viscosity in an accretion disc can be expressed as (Shakura & Sunyaev 1973)

$$\mu_d = \frac{\alpha_d \rho s^2}{\Omega}, \quad (1)$$

where ρ is the density and $s = \sqrt{kT/(\mu m_H)}$ is the isothermal sound speed of gas in the disc, and

$$\Omega(r) = \sqrt{\frac{GM(r)}{r^3}}$$

is the angular frequency of a circular orbit. The dimensionless parameter α_d cannot normally exceed 1 and is generally thought to be $\sim 0.1 - 0.3$ (e.g. Cannizzo 1993). The same parametrization can be applied to the hot gas, i.e. gas at

about the virial temperature, but then it is more appropriate to express the viscosity in terms of the scale height, w ,

$$\mu_h = \alpha_h \rho s w. \quad (2)$$

The cold gas moves on almost circular orbits, so that the time required for gas from radius r to drain to the centre of the disc is roughly (e.g. Shlosman et al 1990)

$$t_d = \frac{r}{v_r} = \frac{r v_{\text{rot}}}{\alpha_d \eta^2 s^2}, \quad (3)$$

where v_r is the radial speed, $v_{\text{rot}} = r\Omega$ is the rotation speed of the gas disc and $\eta = d \ln \Omega / d \ln r$ ($-3/2$ for a keplerian disc).

Since the hot gas is pressure supported, it can drain much faster than a cold disc. However, angular momentum may still prevent it from accreting directly onto a nuclear black hole. For gas at about the virial temperature, the scale height is $w \simeq r$ and the speed of sound is close to the kepler speed, v_{rot} , at the same radius. A rough estimate of the time required to dissipate the angular momentum of the hot gas is then

$$t_{\text{am}} \simeq \frac{r}{\alpha_h v_{\text{rot}}}.$$

For α_h of order unity, this is comparable to the dynamical time, while

$$\frac{t_{\text{am}}}{t_d} \simeq \frac{\alpha_d s^2}{\alpha_h v_{\text{rot}}^2} \ll 1,$$

if the disc gas is cold.

Thus, while the drainage of a cold accretion disc is governed by the dissipation of angular momentum, it is only when hot gas flows inward at speeds approaching the free-fall velocity that we need to be concerned with the effects of rotation on the flow. Dissipation of angular momentum is less of a problem for the accretion of hot gas than it is for cold gas.

For flow speeds comparable to the speed of sound or faster, the angular momentum of the hot gas will be largely conserved, so that its residual angular momentum will cause it to eventually join a disc. We assume that this occurs at a sufficiently small radius to make the drainage time of the disc from the point where the gas joins it point short.

3 FEEDING BY HOT GAS

In the collapse of small protogalaxies, radiative cooling is faster than shock heating, so that gas ends up cold immediately after the collapse (Rees & Ostriker 1977; White & Frenk 1991). In larger systems, which are more tenuous and have higher virial temperatures, some of the gas can form a hot atmosphere after the collapse. The condition for the gas at radius r to be part of a hot atmosphere is that its radiative cooling time be longer than the free-fall time from r to the centre of the protogalaxy. This cooling time is still significantly smaller than the time at which the system collapses, so that the hot gas will start to cool, forming a cooling flow (Fabian 1994), almost immediately after collapse. A central black hole can accrete hot gas from the central region of the cooling flow.

Based on observations of clusters of galaxies, we expect gas taking part in the cooling flow to be sufficiently

inhomogeneous to lead to widespread thermal instability (Nulsen 1986; 1988). The general solution for an inhomogeneous cooling flow is complex, since the flow of each phase must be tracked separately. However, Nulsen (1986) has argued that gas blobs moving relative to the mean flow tend to be rapidly disrupted until they are small enough to be pinned to the mean flow. As a result, the phases tend to flow inward at approximately the same speed, *i.e.* to comove. In the central part of the cooling flow, where conditions change slowly relative to the flow time, we can also expect the flow to be nearly steady.

For the purpose of simulating quasar formation, we take the potentials of galaxies to be exactly isothermal. In that case, the mean temperature of the inhomogeneous gas mixture in a steady, comoving cooling flow will be close to the constant virial temperature, the exact relationship depending on details of the inhomogeneous density distribution in the gas. There is a class of self-similar, inhomogeneous cooling flow solutions in which the mean gas temperature is a constant multiple of the virial temperature, the isothermal cooling flows (Nulsen 1998). The mean gas density and temperature in an isothermal cooling flow are related to the flow time, r/v , by

$$\frac{r}{v} = K \frac{kT}{\mu m_H} \frac{\rho}{n_e n_H \Lambda(T)}, \quad (4)$$

where r is the radius, v the flow velocity, ρ the mean gas density, n_e the mean electron number density and n_H the mean hydrogen number density. T is the effective gas temperature (defined so that the pressure is $\rho kT/(\mu m_H)$) and μm_H is the mean mass per gas particle. The dimensionless constant, K , depends on flow details, including the cooling function and the radial dependence of the mass deposition rate. In clusters the mass flow rate, \dot{M} , is found to be approximately proportional to r (Fabian 1994; Peres et al 1998), which we take to be exact for the purpose of our model. In that case, if the cooling function is approximated by a power law, $\Lambda(T) \propto T^a$, then K depends weakly on the exponent a , ranging from 2.32 to 2.92 for a in the range -0.5 to 0.5 . We adopt the representative value $K = 2.5$ for our calculations.

In an isothermal cooling flow with mass flow rate $\dot{M} \propto r$, the flow velocity is constant, regardless of the details of the cooling function, so that the Mach number is also constant. The arguments below rely on the Mach number of the cooling flow being close to unity initially, since this maximizes the density of the hot gas in the vicinity of a nuclear black hole. This is the most critical aspect of the cooling flow model for quasar formation, since it determines the Bondi accretion rate of the nuclear black hole.

When the Mach number of a cooling flow is low, the linear growth of thermal instability is weak (Balbus & Soker 1989). The inhomogeneous, isothermal cooling flow model relies on non-linear thermal instability (contrary to a common misconception, very small amplitude density fluctuations become non-linear in a cooling flow; Nulsen 1997). However, for Mach numbers of order unity, linear thermal instability is much stronger. If, contrary to our assumptions, the gas is very nearly homogeneous and the thermal instability is weak (or if $\dot{M} \propto r^\eta$ with $\eta < 1$), the Mach number of the cooling flow increases inward and, as it approaches unity, thermal instability becomes strong, causing widespread deposition of cold gas. This tends to make the Mach number

saturate close to one, so that our assumption that the Mach number of the cooling flow in the vicinity of the nucleus is close to one is not sensitive to our assumptions. A cooling flow with a Mach number of order unity is simply a maximal cooling flow. Conveniently, the nuclear accretion rate can be determined exactly for the isothermal cooling flow model (see below), but our quasar formation model is not critically dependent on the assumption that the hot gas forms an isothermal cooling flow. Also note that the gas density in an isothermal cooling flow with Mach number close to one is similar to that obtained by other arguments for the maximum gas density in a protogalaxy (Fall & Rees 1985).

We assume that a nuclear black hole accretes any hot gas coming within its influence by Bondi accretion. At very small r the residual angular momentum causes the accreting gas to pass through a shock (assumed to be radiative) and join an accretion disc. Thus, the final stages of the accretion are still assumed to be through a disc, but we take the nuclear accretion rate to equal the Bondi rate. Thus, the nuclear accretion rate is determined by the density and temperature of the hot gas at the point that the influence of the black hole becomes dominant. There are a number of reasons why this assumption may not be valid, but we have adopted it as the simplest possibility.

The result (4) shows that the steady cooling flow is governed by the requirement that the cooling time equals the flow time, to within factors of order unity. As cooling gas comes under the influence of the black hole, its flow velocity will increase, reducing r/v to the point that cooling is no longer effective. Thus, the transition between the cooling flow and the Bondi solution occurs at about the radius where the initial Mach numbers of the two flows are equal.

The Bondi accretion rate for a monatomic gas (with $\gamma = 5/3$) is (Shu 1991)

$$\dot{M}_h = \pi \rho_i \frac{G^2 M_h^2}{s_i^3}, \quad (5)$$

where ρ_i is the gas density and s_i the adiabatic sound speed at large r , and M_h is the mass of the black hole. Well outside the accretion radius, the density in the Bondi solution is almost constant so that the velocity, v_B , can be determined from the accretion rate, $\dot{M}_h = 4\pi \rho_i v_B r^2$. The gas temperature is also nearly constant, so that Mach number is given approximately by

$$\frac{v_B}{s_i} \simeq \frac{\dot{M}_h}{4\pi \rho_i s_i r^2} = \left(\frac{GM_h}{2s_i^2 r} \right)^2.$$

Equating this to the Mach number, \mathcal{M}_i , of the cooling flow, we find that the Bondi solution takes over at about the radius

$$r_x = \frac{GM_h}{2s_i^2 \mathcal{M}_i^{1/2}}. \quad (6)$$

Using (4) for the gas density and (6) to replace r in the result, we can evaluate the accretion rate (5) as

$$\dot{M}_h = 2\pi K \mathcal{M}_i^{3/2} \frac{kT_i G M_h}{\mu m_H \Lambda(T_i)} \frac{\rho^2}{n_e n_H}, \quad (7)$$

where T_i is the gas temperature at large r (note that the last factor is a constant).

This result is shown to be exact in the Appendix, where T_i is now to be interpreted as the “mean” gas temperature

of a steady isothermal cooling flow and \mathcal{M}_i as its Mach number. The argument in the Appendix also shows that, for the conditions of our model, *i.e.* gas conditions that would give an isothermal cooling flow with $\dot{M} \propto r$, the nuclear accretion rate is independent of the gravitational potential in between the edge of the steady cooling flow and the nucleus. Thus, the accretion rate is largely unaffected by changes to the potential of the galaxy due to deposition of cooled gas. This feature of the model reflects an accidental balance between the competing effects of deepening the potential: a temperature rise, reducing the Bondi accretion rate, and a density increase, tending to increase it.

The result (7) shows that a black hole at the centre of a protogalaxy grows exponentially by Bondi accretion. The timescale for growth is

$$t_{\text{BH}} = \frac{M_{\text{h}}}{\dot{M}_{\text{h}}} = \frac{1}{2\pi K} \frac{n_{\text{e}} n_{\text{H}}}{\rho^2} \frac{\mu m_{\text{H}} \Lambda(T_{\text{i}})}{k T_{\text{i}} G \mathcal{M}_i^{3/2}}, \quad (8)$$

which depends only on the gas temperature and the Mach number in the outer parts of the of the steady cooling flow. Numerically,

$$t_{\text{BH}} \simeq 5.1 \times 10^8 \Lambda_{-23} T_6^{-1} \mathcal{M}_i^{-3/2} \text{y}, \quad (9)$$

where $T_{\text{i}} = 10^6 T_6$ K and $\Lambda(T_{\text{i}}) = 10^{-23} \Lambda_{-23} \text{erg cm}^3 \text{s}^{-1}$.

This can be compared directly with the growth timescale of a black hole accreting at the Eddington rate, t_{Edd} , as

$$t_{\text{BH}} \simeq 11 \Lambda_{-23} T_6^{-1} \mathcal{M}_i^{-3/2} t_{\text{Edd}}.$$

Thus, for the relevant temperatures, the accretion rate from a maximal cooling flow is about one tenth or more of the Eddington rate.

The Bondi radius for a stellar mass object is very small,

$$r_{\text{B}} = \frac{G M_{\text{h}}}{s_{\text{i}}^2} \simeq 6 \times 10^{11} M_{\text{h}} T_6^{-1} \text{cm},$$

for M_{h} in solar masses, so we should be wary of applying our result to accretion onto very small black holes. However, we assume that more massive seed black holes are formed in the cores of all galaxies, as described by Rees (1984), and that these objects then grow to quasars by the accretion of hot gas.

4 QUASAR BIRTH AND DEATH

In order to test the outcome of this quasar formation model, we have incorporated it into a semi-analytical model for galaxy formation, the details of which are described in Nulsen & Fabian (1997) and Nulsen, Barcons & Fabian (1998). In this section we outline modifications we have made to that model in order to track the growth and accretion luminosities of nuclear black holes.

As outlined in the previous section, the major factors that determine the growth rate of a nuclear black hole are the gas temperature and the Mach number of the cooling flow. Expressed in terms of the growth time (8), the time dependence of the mass of a nuclear black hole is given by

$$\ln \frac{M_{\text{h}}(t_2)}{M_{\text{h}}(t_1)} = \int_{t_1}^{t_2} \frac{dt}{t_{\text{BH}}}. \quad (10)$$

The growth time depends on the temperature of the gas, T_{i} , which is determined at the time of collapse of a protogalaxy, and the Mach number, \mathcal{M}_i , of the isothermal cooling flow. The latter factor is the only time dependent part of t_{BH} .

In our galaxy formation model, each collapse produces a dark halo which is taken to be a perfect isothermal sphere (density $\propto r^{-2}$) that is truncated at r_{200} , the radius within which the mean density is 200 times the background density of an Einstein-de Sitter Universe at the time of collapse. The gas temperature produced in the collapse is expressed as

$$T_{\text{i}} = \frac{\mu m_{\text{H}} \sigma^2}{\beta k}, \quad (11)$$

where σ is the line-of-sight velocity dispersion of the halo and β is a dimensionless parameter, generally lying in the range 0.5 to 1, that allows for excess energy in the gas (mostly excess binding energy resulting from supernova driven ejection; $\beta = 1$ corresponds to zero excess energy; β here is determined for each collapse as in the existing semi-analytical model).

The outcome of a collapse is determined by considering a notional, non-radiative collapse. In this collapse, the gas would form a hydrostatic atmosphere, with density proportional to $r^{-2\beta}$ (also truncated at r_{200}). The ratio, τ , of the cooling time to the free-fall time in the notional atmosphere is used to separate the gas into two parts, one with $\tau < \tau_0$ that is cold (due to efficient radiative cooling) immediately after the collapse and one with $\tau > \tau_0$ that forms a hot atmosphere after the collapse. The model parameter τ_0 is of order unity and determines the radius, r_{CF} , in the notional atmosphere that separates these two regions.

Radiative cooling eventually causes hot gas produced in the collapse to cool to low temperatures. As in clusters, the hot gas is little affected by cooling until the age of the system is comparable to its cooling time, at which stage it joins a steady cooling flow before being deposited as cold gas. Thus the hot atmosphere consists of an outer region that is largely unperturbed since the collapse, a transition region, comparable in size to the steady cooling flow, and a central steady cooling flow. The total rate of mass deposition and the extent of the steady cooling flow are determined by the initial state of the hot atmosphere (*e.g.* Fabian & Nulsen 1979). In particular, this means that the Mach number of the steady cooling flow is determined by the initial structure of the hot gas.

Formerly, the hot gas was assumed to cool to low temperature at a time $t = t_{\text{coll}} + t_{\text{cool}}$, where t_{coll} is time of the collapse and

$$t_{\text{cool}}(r) = \frac{3\rho(r)kT_{\text{i}}}{2\mu m_{\text{H}} n_{\text{e}}(r) n_{\text{H}}(r) \Lambda(T_{\text{i}})} \propto r^{-2\beta}$$

is the cooling time of the hot gas in the notional collapse. This made deposition of the cooled gas start discontinuously a short time after collapse. In reality, the onset of the cooling flow will be continuous, commencing immediately after the collapse. To model this, here we assume that the gas cools when

$$t = t_{\text{coll}} + t_{\text{cool}}(r) - t_{\text{cool}}(r_{\text{CF}}),$$

which gives the radius (in the notional atmosphere) of gas that is cooling at time t as

$$r_{\text{cool}} = r_{\text{CF}} \left[1 + \frac{t - t_{\text{coll}}}{t_{\text{cool}}(r_{\text{CF}})} \right]^{1/2\beta}.$$

Since the cooling time of the first hot gas to cool is comparable to the free-fall time (τ_0 about 1), we should expect the initial Mach number of the cooling flow, $\mathcal{M}_{i,0}$, to be about 1. We treat this as a parameter of the model. The time dependence of the Mach number is then determined from the expectation that the flow velocity scales with time as $r_{\text{cool}}/t_{\text{cool}}(r_{\text{cool}}) \propto r_{\text{cool}}^{1-2\beta}$, giving

$$\mathcal{M}_i = \mathcal{M}_{i,0} \left[1 + \frac{t - t_{\text{coll}}}{t_{\text{cool}}(r_{\text{CF}})} \right]^{(1-2\beta)/2\beta}. \quad (12)$$

Using this and (8) in (10) enables us to determine the factor by which the mass of a nuclear black hole grows as a function of the time. Differentiating the result with respect to the time gives the accretion rate and hence the luminosity of the black hole. Of course, the black hole stops growing when the hot gas is exhausted ($r_{\text{cool}} > r_{200}$).

The low level of emission from nuclear black holes in nearby galaxies suggests that the radiative efficiency of accretion is much lower now than it was in quasars (Fabian & Rees 1995; di Matteo et al 1999). Several models, including advection-dominated accretion flows (Narayan & Yi 1995), advection-dominated inflow-outflow solutions (Blandford & Begelman 1999) and other gas processes (Stone, Pringle & Begelman 1999), suggest that this is due to a reduced gas supply. Despite indications that advection-dominated accretion discs do not account for the behaviour of some nearby massive black holes (Di Matteo et al 1999), for the sake of definiteness, we base our model on them. Thus the radiative efficiency is assumed to be high as long as the accretion rate exceeds about $1.3\alpha_d^2 \dot{M}_{\text{Edd}}$ (Esin, McClintock & Narayan 1997). When the accretion rate falls below this value, the radiative efficiency of the nuclear accretion disc is assumed to plummet, in effect turning off emission from the active nucleus. This gives a critical growth time,

$$t_{\text{BH},c} = \frac{M_h}{1.3\alpha_d^2 \dot{M}_{\text{Edd}}} \simeq 3.5 \times 10^7 \alpha_d^{-2} \text{ y}, \quad (13)$$

and when the growth time (8) exceeds $t_{\text{BH},c}$, a quasar turns off, although the nuclear black hole can continue to grow. α_d is treated as a parameter of the model.

Our galaxy formation model uses the block model (Cole & Kaiser 1988) to simulate merger trees. The smallest blocks have a mass of $1.5 \times 10^{10} M_\odot$. In order to simulate the presence of seed black holes, a black hole of (arbitrary) unit mass is associated with each of the smallest blocks. When a block collapses, the black holes associated with all merging sub-blocks are assumed to merge into a single black hole. This makes the mass of a seed black hole proportional to the mass of its halo up to the stage that it starts to grow by Bondi accretion.

A nuclear black hole can only grow by Bondi accretion when it forms in a collapse that produces some hot gas. Such systems are identified with normal galaxies in our model. The model does not allow mergers between normal galaxies, so that normal galaxies only grow in collapses where they accrete dwarf galaxies and gas. Any collapse involving more than one normal galaxy is taken to form a group or cluster of galaxies. Since nuclear black holes are associated with galaxies rather than a group or cluster, we do not track the

growth of black holes for galaxies in these systems. In short, black holes can only grow by Bondi accretion to form quasars in the “normal” galaxies of our simulation. McClure et al (1998) find from optical imaging that most quasars do occur in elliptical, or spheroidal galaxies, so our model should apply well to such objects. It may not be directly relevant to present-day low luminosity Seyfert galaxies, which tend to be in spiral galaxies, but can have undergone a hot-phase era at an earlier stage.

Note that, since the black hole masses are in arbitrary units, the quasar luminosities are too. Adopting a luminosity scale and radiative efficiency for the quasars will fix the scale of the black hole masses. On the bolometric magnitude scale used in the plots in this paper, -15 corresponds to an accretion rate of 10^{-4} black hole mass units per year. If the radiative efficiency is 0.1 and the mass unit is taken as $10^4 M_\odot$, this would correspond to a bolometric luminosity of about $6 \times 10^{45} \text{ erg s}^{-1}$.

5 MODEL RESULTS

For the purpose of the simulations we have taken an open CDM cosmology, with $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, density parameter $\Omega = 0.3$, baryon density parameter $\Omega_b = 0.075$ and $\sigma_8 = 1$.

Because nuclear black holes grow exponentially in our model, the results are quite sensitive to the key parameters, τ_0 , $\mathcal{M}_{i,0}$ and α_d . Using parameter values that favour high growth produces such massive nuclear black holes by the present day that the most recent quasars are inevitably the most luminous. At the other extreme, parameter values can easily be found that result in essentially no growth of the seed black holes. The range of parameters giving substantial, but not excessive, black hole growth is relatively narrow (although it covers a substantial part of the physically reasonable parameter range due to the correlated effects of the parameters). Models presented here are chosen to lie in that range.

Fig. 1 shows distributions of total accretion luminosity for several redshifts for the case $\tau_0 = 1$, $\mathcal{M}_i = 0.9$ and $\alpha_d = 0.1$, while Fig. 2 shows the same thing for $\alpha_d = 0.15$. Since α_d only affects the critical growth time (equation 13), black hole masses and accretion rates are identical in the two models. Differences between them are entirely due to the earlier onset of the ADAF phase for the model of Fig. 2. This effect is greatest at low redshifts, since a greater proportion of the active nuclei is then old and cooling flows in the older collapsed systems have lower Mach numbers (equation 12). A black hole accreting from a cooling flow with a low Mach number has a longer growth time (equation 8) and so is more likely to be an ADAF when α_d is increased. This accounts for the substantial reduction in the numbers of luminous active nuclei at low redshifts between the models of Fig. 1 and Fig. 2.

The model in Fig. 3 is the same as that of Fig. 1, except that the initial Mach number of the cooling flow is $\mathcal{M}_i = 1$. Increasing the Mach numbers of the cooling flows increases the nuclear accretion rate, resulting in greater black hole growth and higher nuclear luminosities. This effect can be seen in Fig. 3, where, for the same seed mass, the most luminous quasars are more numerous at all redshifts.

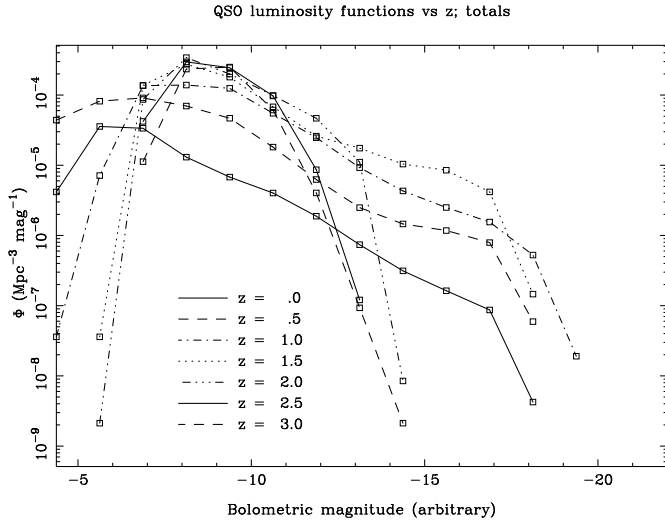


Figure 1. Bolometric luminosity functions for various redshifts. These are for the model with $\tau_0 = 1$, $\mathcal{M}_i = 0.9$ and $\alpha_d = 0.1$. The luminosity used here is the scaled accretion rate in arbitrary units, as described in section 4.

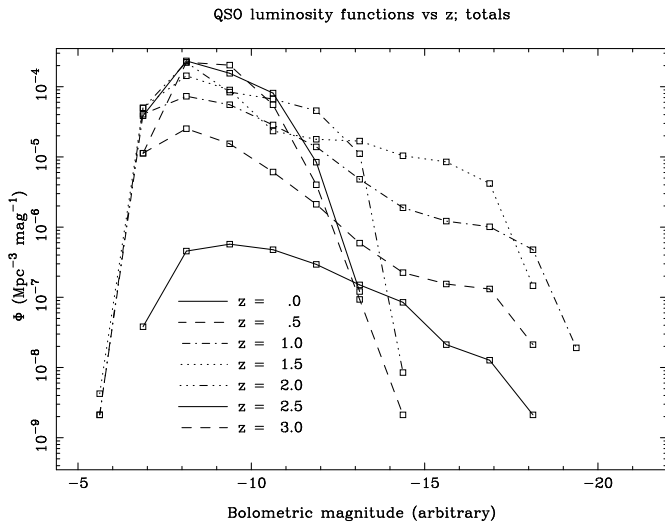


Figure 2. Same as Fig. 1, but for $\alpha_d = 0.15$.

Finally, Fig. 4 shows the bolometric luminosity distributions for a model with $\tau_0 = 0.9$, $\mathcal{M}_i = 1$ and $\alpha_d = 0.1$. This is most readily compared to the model of Fig. 3. The effect of changing τ_0 is more complicated than that of the other two parameters, but its main influence on black hole growth is through the cooling time at the inner edge of the notional hot atmosphere. τ_0 sets the ratio of the cooling time to the free-fall time at r_{CF} , the inner edge of the notional hot atmosphere, so that reducing it reduces the cooling time there, $t_{cool}(r_{CF})$. This cooling time sets the timescale for the evolution of the Mach number of the cooling flow (equation 12). Reducing $t_{cool}(r_{CF})$ causes the Mach number of the cooling flow to decrease more quickly, reducing the overall growth of the nuclear black holes and hence their luminosities.

The bolometric luminosity is, essentially, just the total accretion rate of the black holes and not likely to be a good measure of the visible luminosity of the disc. Despite its

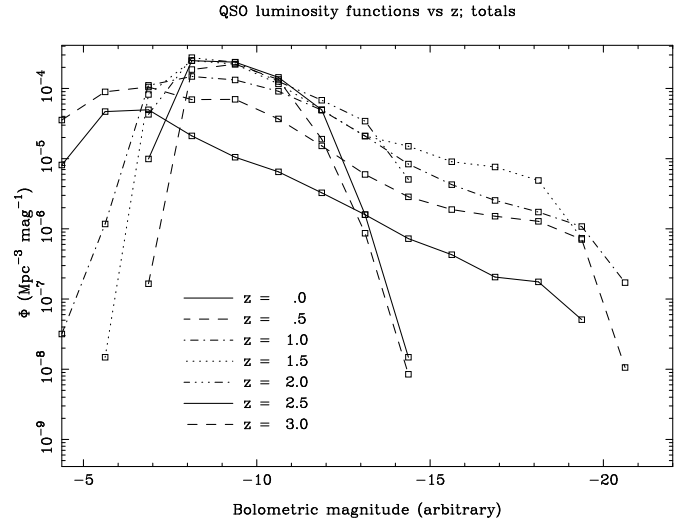


Figure 3. Same as Fig. 1, but for $\mathcal{M}_i = 1$.

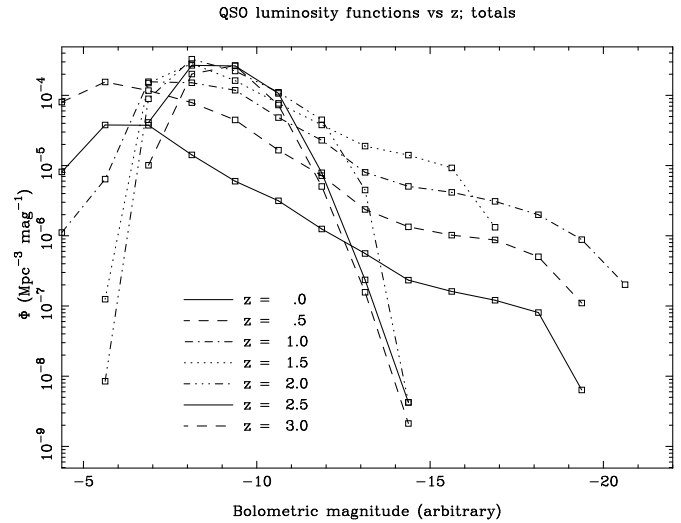


Figure 4. Same as Fig. 3, but for $\tau_0 = 0.9$.

shortcomings, for the sake of definiteness, we use the thermal disc model (Shakura & Sunyaev 1973) to estimate the visible luminosity. This gives the emitted spectrum

$$P_\nu \propto \nu^{1/3} M_h^{2/3} \dot{M}_h^{2/3},$$

for frequency ν . The resulting “visible” luminosity functions are plotted in Fig. 5, for the quasar formation model of Fig. 1 ($\tau_0 = 1$, $\mathcal{M}_i = 0.9$, $\alpha_d = 0.1$).

To compare these to the observed quasar luminosity functions (Boyle et al 1988), we need to convert our arbitrary magnitude scale to absolute blue magnitude. Based on the intensity spectrum of the X-ray background, Fabian & Iwasawa (1999) argue that 85 percent of the accretion power of quasars is absorbed; only ten per cent is seen without some obscuration at 1 keV. If so, then the number densities in Fig. 5 (and preceding Figs) should be reduced by about a factor 10. In that case, adding 2 – 3 to our arbitrary magnitudes to convert to M_B gives rough agreement between our number densities and those of Boyle et al (1988). With

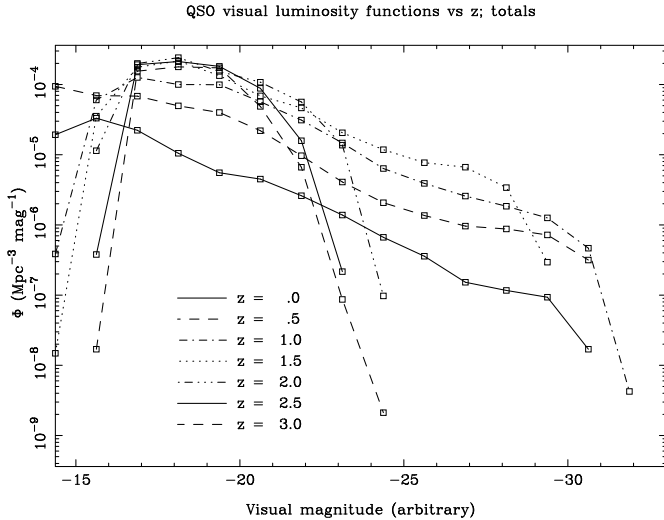


Figure 5. Quasar visible luminosity functions. This shows visible luminosity functions corresponding to the bolometric luminosity functions of Fig. 1, constructed as outlined in the text. The magnitude scale is arbitrary.

this conversion, our luminosity functions are too flat below about $M_B = -27$ and probably too steep above.

The sharp cut offs in the model luminosity functions are largely due to our assumption that the mass of a seed black hole is proportional to the mass of the dark halo in which it forms. A more realistic model for the seed black holes would give them a distribution of masses. In effect, this distribution would be convolved with the luminosity functions, making them fit the observed luminosity functions better.

The redshift dependence of our model is also only in rough agreement with the observed quasar luminosity functions. The greatest discrepancy is the absence in our model of luminous active nuclei at $z = 2$ and earlier. While this is affected to some extent by the collapse model (*i.e.* cosmology), in large part it is due to the time required to grow massive black holes. From equation (8), the growth rate is maximized by minimizing $\Lambda(T_i)/T_i$, which generally means in the hottest collapses. The virial temperature of a halo of mass M collapsing at time t_{coll} scales as $(M/t_{\text{coll}})^{2/3}$, so that, for a given mass, the earliest collapses give the most growth. However, few massive galaxies collapse early, and, since it requires several growth times (equation 9) to produce a massive black hole, very few of these form early in our model.

Fig. 6 is a contour diagram of the distribution of the masses of the nuclear black holes vs spheroid mass at $z = 0$ in the model of Fig. 1. Magorrian et al (1998) and Richstone et al (1998) find that black hole mass is proportional to blue luminosity of the spheroid (bulge). However, there is a substantial spread in this relationship and the data would be consistent with a substantially steeper relationship for the more massive bulges. This is roughly consistent with the ridge and the lower spur at high spheroid mass of the distribution shown here. However, there is no evidence in the data for the extension to high black hole mass for spheroids of about $10^{11} M_\odot$ in the model.

Taking 10^4 black hole mass units to correspond to a spheroid mass of $10^{11} M_\odot$ and using $M_h = 0.005 M_{\text{spheroid}}$

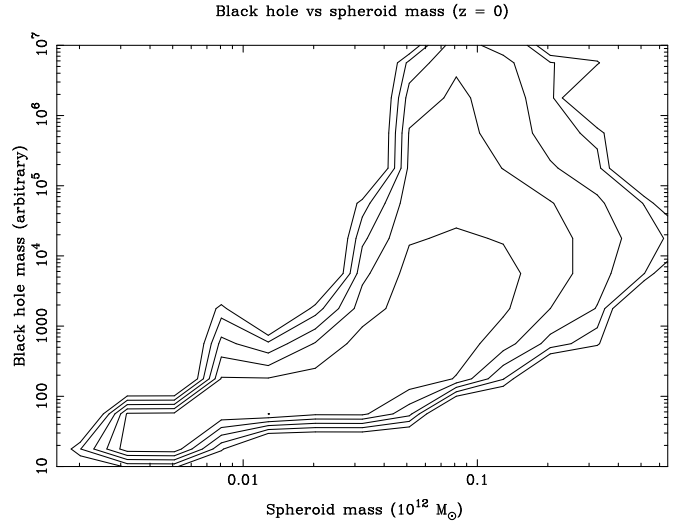


Figure 6. Black hole vs spheroid mass. This is a contour plot of the distribution of the masses of the nuclear black holes (arbitrary units) vs spheroid mass for the model of Fig. 1. Contours are in equal logarithmic intervals.

(Richstone et al 1998) gives the conversion factor of $5 \times 10^4 M_\odot$ per black hole mass unit. In that case, a bolometric magnitude of -15 in the Figs would correspond to a bolometric luminosity of about $3 \times 10^{46} \text{ erg s}^{-1}$.

We can also compare the results of Fig. 1 to the quasar X-ray luminosity function (Miyaji, Hasinger & Schmidt 1998). Using the black hole mass calibration from above and a fixed bolometric correction of about 50 for the 0.5–2 keV X-ray luminosity (Elvis et al 1994; Fabian & Iwasawa 1999) means that a magnitude of -15 in Fig. 1 corresponds to a 0.5–2 keV X-ray luminosity of about $6 \times 10^{44} \text{ erg s}^{-1}$. Assuming that absorption reduces the X-ray luminosity function by a factor of about 10, as above, there is rough agreement between the results in Fig. 1 and the observed X-ray luminosity function of Miyaji et al (1998). However, the fit suffers from essentially the same problems that we found for the visible luminosity function. As in that case, the most serious problem is the lack of high redshift quasars.

6 FEEDBACK EFFECTS

Ciotti & Ostriker (1997) argue that feedback from a quasar will stifle a cooling flow by heating the cooling gas. By raising the gas temperature and reducing its density, this could also dramatically reduce the Bondi accretion rate. However, using Ferland's (1996) CLOUDY and the quasar spectra of Laor et al (1997), we find that the Compton temperature of radio-loud and radio-quiet quasars is approximately one and two million K, respectively. This is cooler than the virial temperature of most of the systems that form quasars. Furthermore, the gas temperature rises inward in the Bondi accretion flow, so that Compton feedback from the quasar is likely, at best, to cool the accreting gas. If Compton cooling is significant, then it will almost certainly increase the nuclear accretion rate, possibly causing it to approach the Eddington limit (Fabian & Crawford 1990).

If Compton cooling reduces the temperature of gas

near to the nucleus, the reduction in pressure will result in inflow at speeds comparable to the sound speed in the uncooled gas. Since the flow within the Bondi radius is already roughly sonic, the accretion rate will not be altered dramatically unless Compton cooling is effective beyond the Bondi radius. Taking the Compton cooling time as $t_C = 3\pi m_e c^2 r^2 / (\sigma_T L_h)$, where m_e is the electron mass, σ_T is the Thomson cross section and L_h is the nuclear luminosity, and taking the flow time from the Bondi radius, $r_B = GM_h/s_i^2$, as $t_{BF} = r_B/s_i$, we have

$$\frac{t_C}{t_{BF}} = \frac{3\pi m_e c^2 GM_h}{\sigma_T L_h s_i} = \frac{3m_e c L_{\text{Edd}}}{4m_H s_i L_h} \simeq 0.8 T_6^{-1/2} \left(\frac{L_h}{L_{\text{Edd}}} \right)^{-1},$$

where the nuclear luminosity has been put in terms of the Eddington luminosity, L_{Edd} . This shows that Compton cooling will be significant outside the Bondi radius for most Eddington-limited active nuclei. In terms of our model, for the accretion rate (7), if the radiative efficiency of the nuclear accretion disc is $\eta = 0.1\eta_{-1}$ (and $K = 2.5$), then

$$\frac{t_C}{t_{BF}} \simeq \frac{m_e \Lambda}{\eta \mathcal{M}_i^{3/2} \sigma_T s_i^3} \frac{n_e n_H}{\rho^2} \simeq 9 \eta_{-1}^{-1} \mathcal{M}_i^{-3/2} T_6^{-3/2} \Lambda_{-23}.$$

We find that the ionization parameter of the gas at the accretion radius, $\xi = L/nr_B^2 \approx 4\pi\eta m_p c^2 s_i \approx 3 \times 10^4$. Under these circumstances, nuclear radiation keeps the gas highly photoionized, so that the effective cooling function is close to pure bremsstrahlung. The Compton cooling time is then less than the infall time for gas temperatures exceeding about 3×10^6 K when the Mach number $\mathcal{M}_i \simeq 1$. This means that the accretion rate may well exceed the Bondi rate in the cases when it would be highest. No allowance has been made for this in our model.

A further effect which may influence some objects is feedback due to radio jets. If the central engine produces jets or outflows which deposit significant energy near the Bondi radius, then the accretion rate can be much reduced. It is not clear how such an effect should be included in our model at this stage. If, as suggested by McLure et al (1998), the radio loud quasars are those with the largest black holes, then feedback from radio jets might be responsible for limiting the growth of the black holes in these systems.

Finally, it has also been suggested that a wind might expel the surrounding gas when quasars becomes sufficiently luminous (Silk & Rees 1998; Fabian 1999). This would lead to a much closer correlation between bulge and remnant mass.

7 DISCUSSION

The implementation of the quasar formation model used here has a number of shortcomings. First, we only follow the growth of black holes in isolated galaxies. This discounts growth in groups and clusters. Because of the high gas temperature, a central galaxy in a group could potentially accrete very rapidly. However, the block model gives no information about the spatial arrangement of collapsing objects, so we are unable to identify central galaxies in groups and clusters. We may therefore be ignoring the most luminous quasars and the most massive black holes.

The truncated isothermal potentials used in the model lead to gas density distributions that are more peaked than

in more realistic collapse models (Navarro, Frenk & White 1997). This affects the time development of the cooling flows, changing the evolution of the Mach number, and so would affect the time dependence of the nuclear accretion rate (equation 7). However, the cooling time of the hot gas is comparable to the free-fall time in normal galaxy collapses, so we should still expect immediate onset of a cooling flow with initial Mach number close to 1 in most cases. The initial cooling time controls the rate of change of the Mach number while it is close to 1 (when the growth rate is largest) and this is comparable to the collapse time. Thus, we do not expect such a change to have a dramatic effect on the results of the simulation. Beyond this, is not clear how a more realistic collapse model would alter our results.

In our simple cooling flow model, the nuclear accretion rate is insensitive to details of the galactic potential. However, this may change in a more realistic model, such as one in which a central star cluster is formed. In that case, matter deposited by the cooling flow beyond the Bondi radius could significantly alter the central potential and so affect the nuclear accretion rate.

The handling of abundances in our galaxy formation model is very crude, only allowing for the effects of Type II supernovae and treating the gas as homogeneous. Increasing the abundance increases the cooling function, hence the growth time (equation 8), and so would reduce black hole masses and quasar luminosities. This may be significant, since the abundances in some quasars appear to be very high (*e.g.* Hamann & Ferland 1993; Ferland et al 1996). On the other hand, the cooling function is considerably less sensitive to abundance for temperatures exceeding about 3×10^6 K and the gas temperature exceeds this value in most of the systems that would be quasars, so we should not expect this to have a major effect on the outcome of the model.

As discussed in section 5, our assumption that the mass of a seed black hole is proportional to the mass of halo in which it resides is too simplistic. Given that Seyfert nuclei can occur in disc galaxies, it seems likely that some active nuclei are not fuelled by hot gas (or, at least, not by gas from a hot halo resulting from the collapse of the protogalaxy). A wide variety of other mechanisms for fuelling active nuclei have been proposed, including starburst activity, interactions between galaxies and the effects of a bar. There is also some cold gas within the region that is effectively drained through a cold accretion disc. Some or all of these gas sources may fuel seed black holes, in which case, they could have a wide range of masses, depending on details of the history of each galaxy. Such effects would be compounded with those due to the processes described in Rees (1984), that are also likely to lead to a range of seed masses.

If a large proportion of active nuclei are heavily absorbed, then mergers between galaxies may affect their luminosity by disturbing the absorbing material, which could alter the luminosity in either direction. In other words, it is possible that much of the evolution of observed optical quasars is due to changes in the obscuring material rather than changes in accretion rate.

The worst shortcoming of our model is its failure to produce quasars at high redshifts. For the cosmological parameters used, a $10^{12} M_\odot$ halo would have collapsed from a 3σ peak at about 10^9 y ($z \simeq 7$). The gas temperature in a halo of mass $10^{12} M_{12} M_\odot$, collapsing at $10^9 t_9$ y, is

$$T \simeq 3.3 \times 10^6 M_{12}^{2/3} t_9^{-2/3} \beta^{-1} \text{ K},$$

for β as defined in equation (11), so that the growth time for a nuclear black hole in such a system would have been

$$t_{\text{BH}} \simeq 1.5 \times 10^8 \beta \mathcal{M}_i^{-3/2} M_{12}^{-2/3} t_9^{2/3} \Lambda_{-23} \text{ y}.$$

The cooling function depends on abundances and ionization (Böhringer & Hensler 1989), but Λ_{-23} lies in the range 0.5 – 2 for the relevant temperatures. Thus, an early collapsing protogalaxy would have had roughly 10 e -folding times to form a massive nuclear black hole before $z \simeq 3$. This shows that the lack of high redshift quasars is not a fundamental shortcoming of our model, although details of the model would clearly need to be modified in order to account for them.

The failure of the model to account for the observed relationship between black hole and spheroid mass (Richstone et al 1998) is due primarily to the exponential black hole growth, which tends to make the larger black holes grow very large. As discussed in the previous section, different feedback mechanisms may enhance or limit the growth rate. If feedback from a radio source limits the growth of the most massive black holes, then this, rather than the fuel supply, might be the cause of the relationship between spheroid and black hole mass. Note that we have ignored many effects in the nuclear accretion disc that could break the simple connection we have assumed between the Bondi accretion rate and the nuclear accretion rate.

Finally, we have attempted to account for the masses of the remnant nuclear black holes and the evolution of the quasars with a single mechanism. If the processes that form the seed black holes account for a substantial part of their mass, or, if other gas sources also play a significant role in the fuelling of active nuclei, then this will not be possible. In that case, accretion of hot gas may simply be one of several fuel sources for quasars. Nevertheless, the results in section 3 show that hot gas is potentially a significant fuel source for AGNs.

Conditions are most favourable for quasar formation in our model when the hot gas supply is greatest, *i.e.* soon after the collapse of a large protogalaxy. In hierarchical collapse models, a collapse will generally include the infall of gas and other galaxies, so that the presence of a close companion of comparable luminosity may be interpreted as an indication of recent collapse. Thus, the results that have been interpreted as showing the gravitational interactions can drive quasars (*e.g.* Bahcall et al 1997) may also be interpreted as indicating that quasars form in systems that have collapsed recently, as expected in the present model. For this purpose, the main difference between the predictions of the models is that no close companion is required in the case that quasars are fuelled by hot gas.

If quasars are fuelled by hot gas, then there should be substantial halos of hot gas around them. Since the gas temperature typically exceeds 3×10^6 K, these should be detectable by their X-ray emission. The limited extent of the hot gas (comparable to the size of the dark halo) makes it hard to separate from the powerful nuclear X-ray emission of a quasar. Nevertheless, the angular resolution of Chandra should be sufficient to detect diffuse emission around some quasars out to redshifts of about 1.

8 CONCLUSIONS

Bondi accretion of the hot gas produced in the collapse of protogalaxies onto a seed population of nuclear black holes is sufficient to form and fuel quasars. A simple simulation shows that this model can account for the optical and X-ray luminosity functions of quasars for $z \lesssim 1.5$, provided that about 90 percent of quasars are obscured. The simulation produces insufficient quasars at high redshifts and predicts a wider range of black hole masses in massive spheroids than has been found.

Hot gas formed in the collapse of large protogalaxies is likely to be the minimum source available for fuelling quasars and so can form the baseline above which other sources contribute. Our model directly confronts and includes problems related to the current fuel supply of massive black holes in elliptical galaxies. The details of the amplitude and evolution of hot gas as a fuel supply is sensitive to the presence of plausible feedback mechanisms, such as heating due to radio jets.

The Bondi accretion rate from the hot gas formed in the collapse of a protogalaxy can exceed the Eddington accretion rate in systems with virial temperatures exceeding about 3×10^6 K, and can be enhanced by feedback due to Compton cooling in such systems. Thus, hot gas is an excellent fuel source for quasars.

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APPENDIX A: BONDI ACCRETION FROM A COOLING FLOW

We consider Bondi accretion by a massive black hole at the centre of a steady inhomogeneous cooling flow. We will show that, for the cooling flow model used here, the accretion rate onto the black hole depends on conditions at the edge of the steady cooling flow, but is insensitive to details of the gravitational potential between there and the central black hole. In particular, when the ratio of specific heats, $\gamma = 5/3$, the accretion rate is completely independent of the intervening potential.

The cooling time of the gas is

$$t_{\text{cool}} = \frac{p}{(\gamma - 1)n_e n_H \Lambda(T)},$$

where p is the pressure, n_e the electron density, n_H the

hydrogen (proton) number density, T the temperature and $\Lambda(T)$ the cooling function. The flow time of the gas is

$$t_{\text{flow}} = \frac{r}{v},$$

where r is the radius and v the flow velocity (positive inward). For the Bondi solution (*e.g.* Shu 1991) at small r , $v \sim r^{-1/2}$, the density varies as $r^{-3/2}$ and the temperature as $r^{-3(\gamma-1)/2}$, so that the ratio of the cooling time to the flow time varies as

$$\frac{t_{\text{cool}}}{t_{\text{flow}}} \sim \frac{T}{\Lambda(T)},$$

Since the temperature increases as r decreases and, for the relevant temperatures and abundances, T/Λ is almost always an increasing function of T , this ratio increases with decreasing r . In a steady cooling flow $t_{\text{cool}} \simeq t_{\text{flow}}$ (Fabian 1994), but as the flow comes under the influence of a central black hole $t_{\text{cool}}/t_{\text{flow}}$ will increase, eventually making cooling negligible. Thus, at sufficiently small r cooling can be ignored and the flow asymptotes to the Bondi solution.

We begin by outlining the Bondi solution. For a steady, spherical flow, the mass flow rate (inward) is

$$\dot{M} = 4\pi\rho v r^2 = \text{constant},$$

where ρ is the gas density. The flow is adiabatic, so that

$$\frac{T}{\rho^{\gamma-1}} = \frac{T_0}{\rho_0^{\gamma-1}},$$

where T_0 and ρ_0 are evaluated a long way from the black hole. Using these, we can express the flow speed as

$$v = \frac{\dot{M}}{4\pi\rho_0 r^2} \left(\frac{T_0}{T} \right)^{1/(\gamma-1)}. \quad (\text{A1})$$

Bernoulli's theorem gives

$$H + \frac{1}{2}v^2 - \frac{GM}{r} = H_0, \quad (\text{A2})$$

where r is the radius, $H = \gamma p/[(\gamma - 1)\rho]$ is the specific enthalpy of the gas, H_0 is the specific enthalpy at temperature T_0 and M is the mass of the black hole. Differentiating (A2) with respect to r and using (A1) to find dv/dr gives

$$\left(H - \frac{v^2}{\gamma - 1} \right) \frac{1}{T} \frac{dT}{dr} = \frac{2v^2}{r} - \frac{GM}{r^2},$$

so that at the sonic point, r_s , where $v_s^2 = (\gamma - 1)H_s$, we must have

$$v_s^2 = \frac{GM}{2r_s}.$$

Using these results in (A2) gives

$$\frac{GM}{r_s} = \frac{4(\gamma - 1)}{5 - 3\gamma} H_0,$$

enabling us to evaluate all quantities at the sonic point in terms of T_0 , ρ_0 and M . Evaluating the mass flow rate at the sonic point then gives the Bondi accretion rate as

$$\dot{M}_h = \pi\rho_0 \frac{(GM)^2}{s_0^3} \left(\frac{5 - 3\gamma}{2} \right)^{\frac{3\gamma-5}{2(\gamma-1)}}, \quad (\text{A3})$$

where $s_0 = \sqrt{\gamma p_0/\rho_0}$ is the speed of sound in gas a long way from the black hole. The last factor in (A3),

$$q(\gamma) = \left(\frac{5-3\gamma}{2} \right)^{\frac{3\gamma-5}{2(\gamma-1)}},$$

is finite for both $\gamma \rightarrow 1$, when $q \rightarrow e^{1.5}$, and $\gamma \rightarrow 5/3$, when $q \rightarrow 1$.

Now consider a cooling flow. Thermal instability causes an inhomogeneous cooling flow to deposit gas throughout the flow at a rate that is conveniently expressed as (Nulsen 1986, 1988; values there assume $\gamma = 5/3$)

$$\xi \frac{(\gamma-1)\rho R}{\gamma p},$$

where R is the power radiated per unit volume by the gas, ρ is now the mean density of the gas and ξ is dimensionless. The value of ξ depends on details of the density distribution and physical behaviour of the inhomogeneous gas, and is typically of order unity. We treat it as a constant parameter in what follows, since this leads to the simplest physically useful flow models (this approximation is exact for the isothermal cooling flow models; Nulsen 1998). The mass conservation equation for a steady flow is then

$$\frac{1}{r^2} \frac{d}{dr} \rho v r^2 = \xi \frac{(\gamma-1)\rho R}{\gamma p}. \quad (\text{A4})$$

The corresponding energy equation is (Nulsen 1988)

$$\frac{1}{\gamma-1} p v \frac{d}{dr} \ln \Sigma = (1-\xi) R, \quad (\text{A5})$$

where $\Sigma = T/\rho^{\gamma-1}$, with T being the temperature corresponding to the mean density (ρ) at pressure p . Σ determines the effective entropy of the inhomogeneous gas mixture. Eliminating R between these two equations and integrating gives

$$\frac{\Sigma}{\Sigma_i} = \left(\frac{\dot{M}}{\dot{M}_i} \right)^{\gamma(1-\xi)/\xi}, \quad (\text{A6})$$

where \dot{M}_i and Σ_i are evaluated at r_i , a fixed point in the cooling flow, which we will take to be at the outer edge of the steady flow, well outside the region that is perturbed by the black hole. Thus, the cooling flow model enforces a fixed relationship between the entropy and \dot{M} throughout the steady flow.

The derivation of equation (A6) makes no reference to the potential (or the momentum equation) and it applies at small r , in the Bondi flow where cooling is negligible. The quantities ρ_0 and T_0 (*i.e.* s_0) in the Bondi accretion rate (A3) are no longer well defined, but, since it is constant, the entropy, $\Sigma_0 = T_0/\rho_0^{\gamma-1}$, is. Furthermore, (A6) requires

$$\frac{\Sigma_0}{\Sigma_i} = \left(\frac{\dot{M}_h}{\dot{M}_i} \right)^{\gamma(1-\xi)/\xi},$$

giving

$$\rho_0 = \left(\frac{T_0}{\Sigma_i} \right)^{\frac{1}{\gamma-1}} \left(\frac{\dot{M}_h}{\dot{M}_i} \right)^{-\frac{\gamma(1-\xi)}{\xi(\gamma-1)}}.$$

We use this to eliminate ρ_0 in (A3), then solve the resulting equation for \dot{M}_h and put $\Sigma_i = T_i/\rho_i^{\gamma-1}$, where T_i and ρ_i are the temperature and density at r_i . After some algebra this gives

$$\dot{M}_h = \left[\pi \rho_i \frac{(GM)^2}{s_i^3} q(\gamma) \right]^{\kappa} \dot{M}_i^{1-\kappa} \left(\frac{T_0}{T_i} \right)^{\frac{\kappa(5-3\gamma)}{2(\gamma-1)}}, \quad (\text{A7})$$

where $\kappa = \xi(\gamma-1)/(\gamma-\xi)$.

This argument makes no reference to the gravitational potential in which the cooling flow takes place, but \dot{M}_h depends on details of the potential in two ways. First, gas properties at the edge of the cooling flow are affected to some extent by the potential (\dot{M}_i is governed largely by initial conditions in a collapse). Generally, we can assume that any disturbance to the potential is at $r \ll r_i$ and has little effect on the cooling flow at r_i . The second means of influence is through T_0 , which is still not well defined. Despite this, we can expect T_0 to be comparable to the gas temperature at the point where the black hole starts to have an appreciable affect on the cooling flow. Since the influence of the black hole is felt outside the sonic point, this temperature will be comparable to the “virial” temperature at that place and, unless the galaxy potential is strongly non-isothermal, we can expect it to be comparable to T_i . In general, the influence of the potential on \dot{M}_h is weak.

For the case of interest, $\gamma = 5/3$ and \dot{M}_h does not depend on T_0 , but is determined completely by the mass of the black hole and the gas properties near the outer edge of the cooling flow. This simple result comes about because, for $\gamma = 5/3$, the Bondi accretion rate (A3) depends on the gas properties through the entropy alone ($\rho_0 s_0^{-3} \propto \Sigma_0^{-3/2}$), so that it may be regarded as specifying a relationship between Σ and \dot{M} . The requirement (A6) of the cooling flow specifies a second relationship between Σ and \dot{M} which is only satisfied simultaneously by a unique \dot{M} .

For an inhomogeneous isothermal cooling flow, if $\dot{M} \propto r^\eta$, then equations (A4) and (A5) require that

$$\xi = \frac{2\gamma\eta}{3(\gamma-1) + \eta(\gamma+1)},$$

making

$$\kappa = \frac{2\eta}{3+\eta}.$$

We take $\gamma = 5/3$ for the remainder of this section.

For an isothermal cooling flow, Nulsen (1998) has shown that

$$\rho_i = Q \frac{3-\eta}{2} \left(\frac{\rho^2}{n_e n_H} \right) \frac{v_i k T_i}{\mu m_H r_i \Lambda(T_i)},$$

where v_i the flow speed at r_i and $\rho^2/(n_e n_H)$ is constant for the temperatures of interest. The factor Q is a constant that depends on details of the cooling function. For power laws, $\Lambda(T) \propto T^a$, with a in the range $[-0.5, 0.5]$, Q ranges from 2.32 to 2.93. Since $d \ln \Lambda / dT$ lies in about this range for the temperatures of interest, we use the representative value $Q = 2.5$ in all models. Using the expression for ρ_i to eliminate the density in $\dot{M}_i = 4\pi \rho_i v_i r_i^2$ and using the result to replace \dot{M}_i in (A7) gives, after some further algebra,

$$\dot{M}_h = \frac{(3-\eta)\pi Q k T_i G M \mathcal{M}_i^{\frac{3}{2}}}{\mu m_H \Lambda(T_i)} \frac{\rho^2}{n_e n_H} \left(\frac{2r_i s_i^2 \mathcal{M}_i^{\frac{1}{2}}}{GM} \right)^{\frac{3(1-\eta)}{3+\eta}}, \quad (\text{A8})$$

where $\mathcal{M}_i = v_i/s_i$ is the Mach number at r_i .

Observations show that $\eta \simeq 1$ in the well studied cluster cooling flows (Fabian 1994; Peres et al 1998) and this is the value that we use for our models, giving

$$\dot{M}_h = \frac{2\pi Q k T_i G M \mathcal{M}_i^{3/2}}{\mu m_H \Lambda(T_i)} \left(\frac{\rho^2}{n_e n_H} \right). \quad (\text{A9})$$

We note that η is not well determined and, since the factor in parentheses in (A8) is usually large, a small change in η can have a large effect on \dot{M}_h . For the isothermal cooling flow model, the radial dependence of the Mach number is $\mathcal{M} \propto r^{(\eta-1)/2}$, so that \mathcal{M} is constant in our models. If $\eta < 1$ ($\eta > 1$), then \mathcal{M} increases (decreases) inward. In general, thermal instability in a cooling flow is greater when the Mach number is larger (Balbus & Soker 1989), which will tend to limit the rise in the Mach number for $\eta < 1$. This limits the error in \dot{M}_h in that case. For $\eta > 1$ there is no limiting effect, so that, if $\eta > 1$, (A9) could substantially overestimate the nuclear accretion rate. If so, the rate of accretion of hot gas would be insufficient to account for quasars.